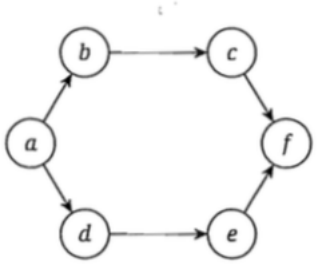
CMP741 Exercises for Week 1

10 February 2022

1. Write all topological orderings of the graph below.



2. Give an algorithm to detect whether a given undirected graph contains a cycle. If the graph contains a cycle, your algorithm should output one. The running time of your algorithm should be O(m+n) for a graph with n vertices (nodes) and m edges.

3. Suppose that we are given a directed graph that may or may not be a DAG. Extend the topological algorithm (Section 3.6 in K-T) so that it outputs a) either a topological ordering of G, thus deciding that it is a DAG, b) or finding a (directed) cycle, thus showing that G is not a DAG. The running time of your algorithm should be O(m+n) for a directed graph with n vertices (nodes) and m edges.

4. A binary tree is a rooted tree in which each node has at most two children. Show by induction that in any binary tree the number of vertices (nodes) with two children is exactly one less than the number of leaves. (Can you think of a direct method to show this?)